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On the Choice of Strain Measures in Geomechanics

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Abstract: In the process of understanding and developing constitutive models for geomaterials, the stress-strain behaviour of the test material is commonly studied by performing traditional triaxial compression tests and occasionally true triaxial tests. In both cases, the Cauchy, or true stress measure, is easily adopted, but when it comes to selecting a suitable strain measure, that expresses the relative deformation of the continuum, it becomes more difficult. Three methods for the analysis of triaxial and true triaxial tests are examined in this paper. The first method correspond to the conventional analysis of triaxial tests. The results of this paper will reveal that the method uses the theory of infinitesimal deformations inconsistently, as a finite strain measure is mixed with an infinitesimal strain measure. The second method uses the theory of infinitesimal deformations more consistently. However, application of this theoretically correct method produces erroneous results under certain conditions. The produced error on the principal strains is insignificant for all practical purposes, but becomes significant when it comes to evaluating the volumetric strain. The error on the volumetric strain can be eliminated by using a non-linear and finite strain measure. The third method is based on the natural strain measure, which is both non-linear and finite. To pinpoint the problem associated with these methods, errors introduced in the traditional analysis of triaxial and true triaxial tests are calculated and evaluated. The effect of the three methods on the prediction of stress-strain curves is subsequently examined using an advanced constitutive model to predict the soil response in a conventional triaxial compression test.

1 INTRODUCTION

Traditional triaxial tests, drained and undrained, are commonly used in the study of the stress-strain behaviour of geomaterials. Drained tests are solely considered here, but all observations presented in this paper apply to the undrained case as well. During drained triaxial tests coherent values of axial displacement, volume change, confining pressure and axial load are measured. Since all directional measurements coincide with the principal axes of stresses and strains, the analysis of the test data ought to be straight forward. The stress and strain measures must according to Malvern (1969) be work conjugate and, furthermore, refer to the same configuration (reference or current) when constitutive relations are investigated. From an engineering point of view, it is obvious to use the Cauchy or true stress, as stress measure. This is adopted throughout this paper. The Cauchy stress can in simple terms be expressed as the ratio between current load and current area (Crisfield, 1991) and can with ease be calculated from the measurements carried out during a triaxial test. In cases where strains and displacements are assumed infinitesimal the distinction between a description based on a reference or a current configuration becomes arbitrary as all stress and strain measures are work conjugate in this case. Consequently, the engineering strain measure is work conjugate with the Cauchy stress under this assumption. In situations where displacements or strains are large, another strain measure, a finite strain measure, must be introduced. This measure must be work conjugate to the Cauchy stress measure. The natural strain increment, as stated in Abaqus (1995) and Crisfield (1991), satisfy this requirement. Both strain measures the natural strain increment and the engineering strain are adopted in this paper. In the traditional analysis of triaxial tests (denoted by T) the axial strain is calculated as the ratio between the measured axial displacement and the initial height of the specimen, e.i. the engineering strain measure or the infinitesimal strain measure is used. Products of displacement derivatives are neglected in the theory of infinitesimal deformations (Spencer, 1980). The volumetric strain is traditionally calculated as the ratio between measured volume change and the initial volume of the specimen. Squares and products of displacement derivatives are not neglected in this calculation (Spencer, 1980). Hence, a finite strain measure is adopted and an inconsistency arises in the assumptions as finite and infinite strain measures are mixed. This inconsistency is not limited to triaxial tests and becomes even more evident when analysing true triaxial tests, where the volume change is measured together with the displacements in the three principal directions. The inconsistency can be eliminated by following one of two distinct methods, either by adopting the natural strain increment or simply by adopting the engineering strain consistently in the analysis. Using these two methods denoted by N and E, respectively, requires a computation of an exact displacement field before the strains can be calculated. Both methods are in the following illustrated for the triaxial and the true triaxial case and the results are compared with the traditional method T. The effect of the three methods on some key geotechnical parameters is investigated together with the effect on modelling the stress-strain behaviour using an advanced constitutive model.

2 ANALYSIS OF TRIAXIAL TESTS

During a drained triaxial test, coherent values of axial displacement, volumetric change, axial load and confining pressure are measured. On this basis, it is possible to obtain the radial displacement and true axial stress, thus yielding a complete stress-strain description of the soil specimen under axissymmetric conditions. The radial displacement can only be calculated under the assumption that the radial and angular strain equals which as stated by Kirkpatrick & Belshaw (1968) is the case for all practical purposes.

2.1 *Analysis based on the exact displacement field*

In a triaxial test the deformation of the soil specimen is characterised by the compression or elongation in the axial and radial directions.

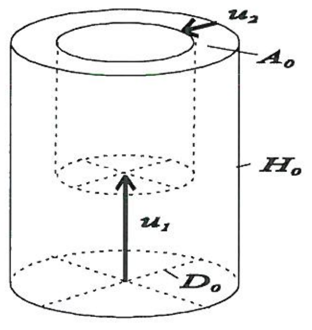

Fig. 1. Definition of geometric quantities.

The original size of a sample is fully described by the initial height, H_0, and the initial diameter, D_0, whereas the size in a deformed stage is fully described by its original size and the displacement components u_1 and u_2. Compression, as shown in figure 1, is considered positive. The current height, H, diameter, D, and cross-sectional area, A, are given by:

$$H = H_0 - u_1; \quad D = D_0 - 2u_2 \tag{1}$$

$$A = \frac{\pi}{4}(D_0 - 2u_2)^2 = \frac{V_0 - \Delta V}{H_0 - u_1} \tag{2}$$

The current cross-sectional area is traditionally used in the calculation of the axial stress. Therefore, it follows that the axial stress is of the Cauchy type. The volume change, ΔV, is of great importance in geomechanics. In terms of the displacement components it may be expressed as:

$$\Delta V = \frac{\pi}{4}\left[H_0 D_0^2 - (H_0 - u_1)(D_0 - 2u_2)^2\right] \tag{3}$$

The radial displacement is traditionally not calculated in the analysis of triaxial test, but this quantity is indispensable for a complete description of the displacement field. As the axial displacement and the volume change are measured directly, the radial displacement can be expressed as:

$$u_2 = \frac{D_0}{2} - \sqrt{\frac{V_0 - \Delta V}{\pi(H_0 - u_1)}} \tag{4}$$

The radial stress or the confining pressure is measured directly as a Cauchy type of stress. Therefore, no intermediate calculations are required. An exact representation of the displacement and the Cauchy type of stress field has now been set-up.

The determination of geotechnical design parameters and the general study of material behaviour are commonly based on element tests, such as the triaxial test. As initial sample dimensions may vary from sample to sample and from apparatus to apparatus, the relevant parameters can obviously not be based on displacements. The parameters has to be based on relative deformations. There are two strain measures that are work conjugate to the Cauchy type of stress measure. That is, as stated in Malvern (1969), the engineering strain and the natural strain increment. Both strains measures are adopted in this paper.

2.2 *Engineering strain vs. natural strain*

The linear engineering strain measure and the non-linear natural strain measure are briefly discussed in order to indicate their use and limitations. The simplest definition is the engineering strain, which is traditionally used in the theory of infinitesimal deformations:

$$\varepsilon_1^E = \frac{u_1}{H_0};\ \varepsilon_2^E = 2\frac{u_2}{D_0} \tag{5}$$

The natural strain increment is often employed in the theory of finite deformations and/or in the theory of plasticity (Abaqus, 1995b). The natural strain increment is closely associated with the natural strain and based on the ratio between the initial height, H_0, and the initial diameter, D_0, and the current quantities, respectively:

$$\varepsilon_1^N = \ln\left(\frac{H_0}{H_0 - u_1}\right);\ \varepsilon_2^N = \ln\left(\frac{D_0}{D_0 - 2u_2}\right) \tag{6}$$

The natural strain measure makes no distinction between initial and final quantity and an interchange merely changes the sign. The difference between the natural and engineering strain measures in the one dimensional case is illustrated in figure 2.

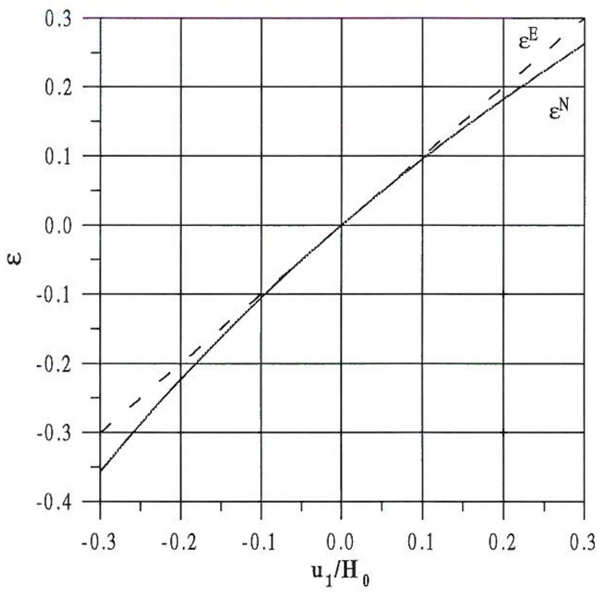

Fig. 2. The strain measures ε_1^E and ε_1^N for the one dimensional case.

It is seen that $\varepsilon_1^E > \varepsilon_1^N$ and that the deviation is in of the order 4-5 % for $|u_1/H_0| < 0.1$. The deviation is in general accepted and the assumption of infinitesimal deformations is commonly assumed to be valid. However, the difference becomes more pronounced when it comes to calculating the volumetric strain.

The volumetric strain is within the framework of the theory of infinitesimal deformations defined as the sum of the principal strains:

$$\varepsilon_v^E = \varepsilon_1^E + 2\varepsilon_2^E \tag{7}$$

The volumetric strain is traditionally calculated as the ratio between the measured volume change and the initial volume of the specimen:

$$\varepsilon_v^T = \frac{\Delta V}{V_0} \tag{8}$$

Within the normal range of deformations, the disparity between the methods may exceed 15-20%. The volumetric strain, based on the natural strain definition, is found by addition of the principal strains:

$$\varepsilon_v^N = \varepsilon_1^N + 2\varepsilon_2^N = \ln\left(\frac{V_0}{V_0 - \Delta V}\right) \tag{9}$$

The disparity between the expressions in (7) and (9) may exceed 15-20% within the normal range of deformations.

Whether the engineering or the natural strain measure is chosen in the analysis of triaxial tests, depends on whether finite or infinite deformations apply to the geotechnical problem under investigation. The form of the constitutive relation must, moreover, be considered (Abaqus 1995b). The volumetric strain measures is more thoroughly discussed in the succeeding section.

2.3 *Analysis based on strains*

The traditional analysis of triaxial tests, denoted by T, is strain wise performed by using the expressions in (5) and (8). An analysis based solely on the theory of infinitesimal deformations (method E) is on the other hand performed by using the expressions in (5) and (7), whereas the method denoted by N uses the expressions in (6) and (9). The traditionally analysis of triaxial tests leads to an inconsistently use of the theory of infinitesimal deformations, as a finite strain measure (8) is mixed with an infinite measure (5).

The two proposed methods, N and E, uses their theoretical background consistently and could therefore both be used in the analysis of triaxial tests. However, method E has some limitations as significantly errors are introduced under certain conditions. The error is investigated in the following and it is shown how the use of the natural strain measure leads to an exact description of the deformations.

2.3.1 Errors produced using E and T

The radial displacement component, u_2, can by using the expressions in (5), (7) and (8) be expressed as:

$$u_2^{T,E} = \frac{4\Delta V - \pi D_0^2 u_1}{4\pi D_0 H_0} \tag{10}$$

A comparison of (4) and (10) reveals the effect of the linear approximation. The error, e, on the radial displacement component is given by (11) and is shown in figure 3.

$$e = u_2 - u_2^{T,E} = \frac{D_0}{2}\left(1 - \frac{4\Delta V - \pi D_0^2 u_1}{8V_0}\right) - \sqrt{\frac{V_0 - \Delta V}{\pi(H_0 - u_1)}} \tag{11}$$

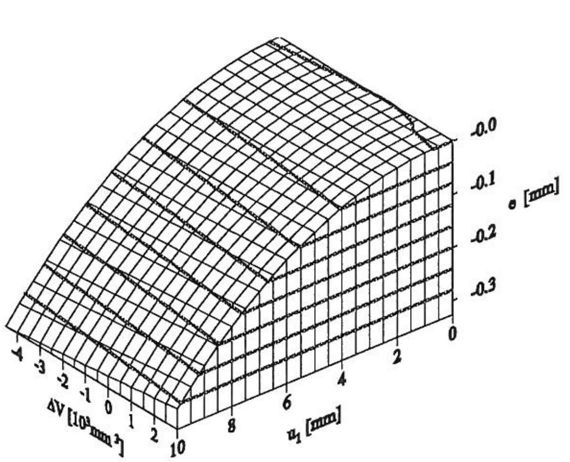

Fig. 3. Error due to the use of method E, H_0=D_0=70 mm.

The use of method E can at moderate to high levels of deformation produce significantly errors. So the method E should only be applied in cases where deformations are truly small.

2.3.2 Natural strain

If the natural strain definition is applied, the radial deformation can be determined on the basis of (6) and (9):

$$u_2^N = \frac{D_0}{2} - \sqrt{\frac{V_0 - \Delta V}{\pi(H_0 - u_1)}} \tag{12}$$

As this expression is seen to be identical to (4), it appears that the error e, and that the inconsistency caused by mixed finite and infinite strain measures can be eliminated by adopting the natural strain.

3 ANALYSIS OF TRUE TRIAXIAL TESTS

During the true triaxial test, coherent values of principal displacements and stresses as well as the volume change are measured. A deformed and undeformed cubical specimen is shown in figure 4.

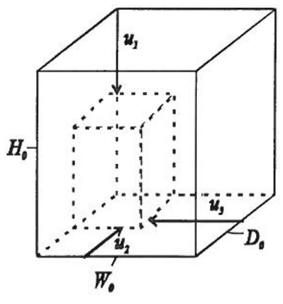

Fig. 4. Definition of geometric quantities.

The initial height, width, and depth of the cube is denoted by H_0, W_0 and D_0, respectively. The matching displacements are denoted by u with subscripts, 1 to 3 and define the current dimensions of the specimen:

$$H = H_0 - u_1;\ W = W_0 - u_2;\ D = D_0 - u_3 \tag{13}$$

Since all displacements are measured in the principal directions, it is possible to evaluate the influence of the two strain measures on the volume change, as the volume change is measured directly.

It could be argued that the objective is irrelevant since all deformation quantities are measured and any inconsistency should therefore automatically be revealed. However, on several of the devices used for true triaxial testing, only two directional displacements are measured together with the volume change.

Our experience in this area suggest that all deformations ought to be monitored as inconsistencies between the three displacement measurements and the measured volume change will otherwise occur. The correct volume change, ΔV, of the cubical specimen can in terms of displacement components be expressed as:

$$\Delta V = W_0 H_0 D_0 - (H_0 - u_1)(W_0 - u_2)(D_0 - u_3) \tag{14}$$

3.1 *Engineering strain*

Assuming that the theory of infinitesimal deformations is valid, the engineering strains are given by:

$$\varepsilon_1^E = \frac{u_1}{H_0};\ \varepsilon_2^E = \frac{u_2}{W_0};\ \varepsilon_3^E = \frac{u_3}{D_0} \tag{15}$$

The volume change can under the same assumption be expressed as:

$$\Delta V = \left(\frac{u_1}{H_0} + \frac{u_2}{W_0} + \frac{u_3}{D_0}\right) W_0 H_0 D_0 \tag{16}$$

Comparison of (14) and (16) reveals that the error, *e,* on the volume change is:

$$e = u_1 u_2 u_3 - u_1 u_2 D_0 - u_1 u_3 W_0 - u_2 u_3 H_0 \tag{17}$$

Thus, the error arises due to the negligence of higher order terms in the displacement expressions, which indeed form the basis of the infinitesimal displacement theory. Assuming a plane strain condition (u_3=0) and setting height, depth and width equal to 70 mm, the error normalised with respect to D_0 can be plotted as a function of u_1 and u_2. The error function is shown in figure 5.

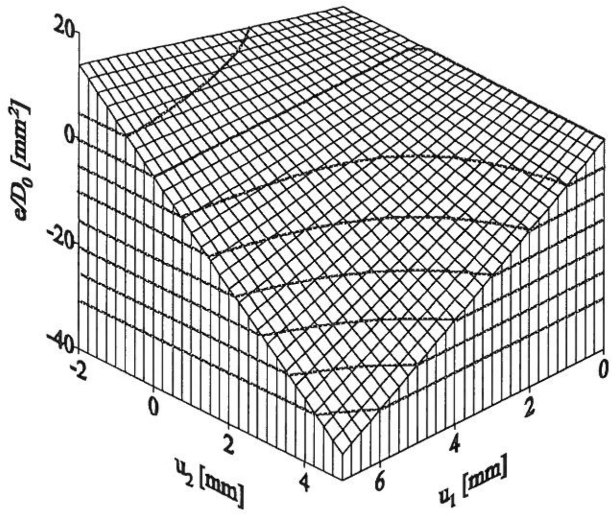

Fig. 5. Error due to the use of method E under the assumption of plain strain conditions.

The figure shows that the error on the volume change becomes quite important even though the error on the principal strains was insignificant for all practical purposes, section 2.2.

3.2 *Natural strain*

Applying the natural strain definition instead of the engineering strain definition can be shown to produce an exact representation of the volume change. This was also the result obtained in the analysis of triaxial tests.

4 EFFECT ON SOME KEY GEOTECHNICAL PARAMETERS

As described in the preceding sections, precipitate analysis of both triaxial and true triaxial tests can lead to erroneous results. How the strain measures affects the geotechnical parameters, is examined in the following. First by performing a simple analysis of a conventional triaxial compression test and secondly by calibrating a constitutive model which may be applied in more complex boundary value problems.

The conventional triaxial test is performed on Eastern Scheldt Sand deposit with a relative density of D_r=0.70. The initial sample size was measured to H_0=71.5 mm and D_0=69.5 mm. More details concerning the sand and test procedures are found in Jakobsen and Praastrup (1998).

4.1 *Analysis of a conventional triaxial test*

The effect of different strain measures, on the traditional geotechnical design parameters, is illustrated by an analysis of a conventional triaxial test. The analysis is performed in two parts, firstly following an analysis based on the exact displacement field and secondly by the three methods denoted by T, N and E.

The specimen was at first isotropically consolidated to an isotropic state of stress of 160 kPa and secondly sheared at a constant confining pressure of σ'_3=160 kPa. Figure 6 shows the deviator stress $q=\sigma_1-\sigma_3$ versus the directly measured axial displacement u_1.

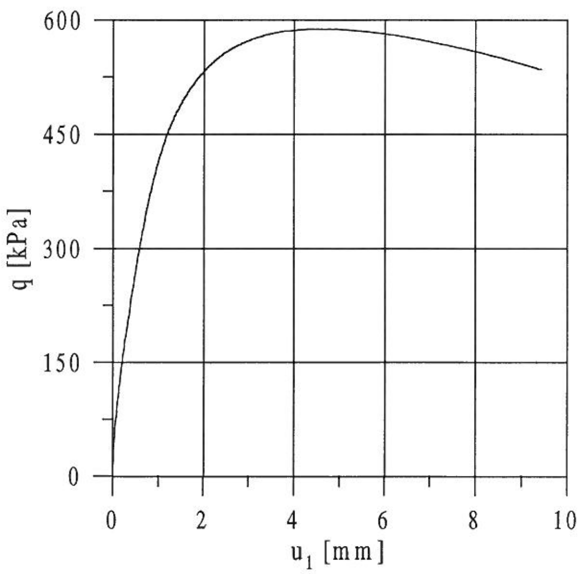

Fig. 6. Deviator stress versus axial displacement.

The graph shows a typical stress-strain curve for medium dense sand, performed under the above mentioned stress levels. The initial slope of the stress-strain curve is steep and flattens out as the specimen hardens until failure and progresses into softening hereafter.

The cross-sectional area of the specimen determines, indirectly, the axial stress applied onto the specimen during shear. As the calculation of the cross-sectional area in each of the three methods are identical, the axial stress remains unaffected by the applied methods. The radial stress is in triaxial tests measured directly and is therefore unaffected by the methods. Geotechnical parameters that solely depends on stresses are hence unaffected of the three methods. Therefore, the two strain measures do not affect the geotechnical parameters that are determined solely on the basis of the stresses. The friction angle φ' is an important geotechnical parameter that is solely based on stresses. This parameter is unaffected by the strain measure. The secant friction angle for the test shown in figure 6 is φ'=40.3°. Geotechnical parameters that are based solely on strains, or both stresses and strains, will on the contrary be affected by the method and the strain measure used. Figure 7 shows the measured volume change versus the measured axial displacement.

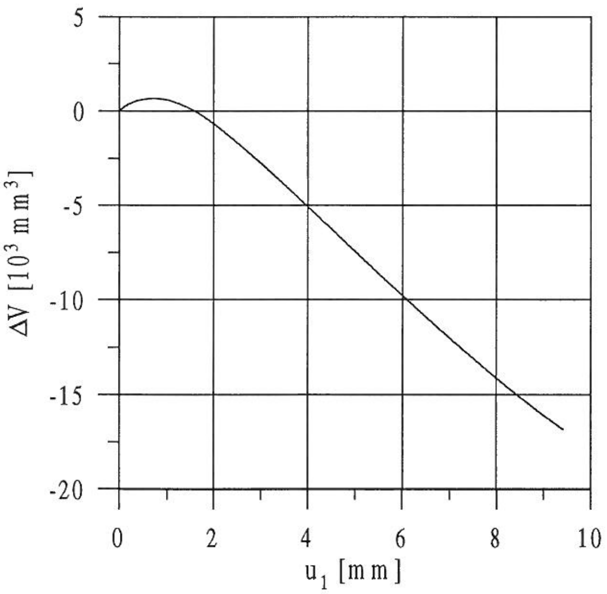

Fig. 7. Volume change versus axial displacement.

The figure shows that the specimen initially compresses and dilates subsequently. The effect on a particular strain dependent parameter depends on how the parameter is determined and in particular on the strain level. Parameters determined at low strain levels are less affected by the chosen method than parameters determined at high strain levels.

The initial tangent modulus of a stress-strain curve is for practical purposes unaffected as the parameter is determined in the beginning of the shearing process, (Janbu 1963). Geotechnical parameters such as the characteristic angle and angle of dilation are affected more significantly. Parameters determined by strain increments will however be less affected than parameters determined by total strains.

In section 2.1 it was postulated that the volumetric behaviour is greatly influenced by the choice of strain measure. This effect is illustrated in figure 8, where the volumetric behaviour of the specimen is presented in terms of strains and plotted versus both the axial engineering and natural strains.

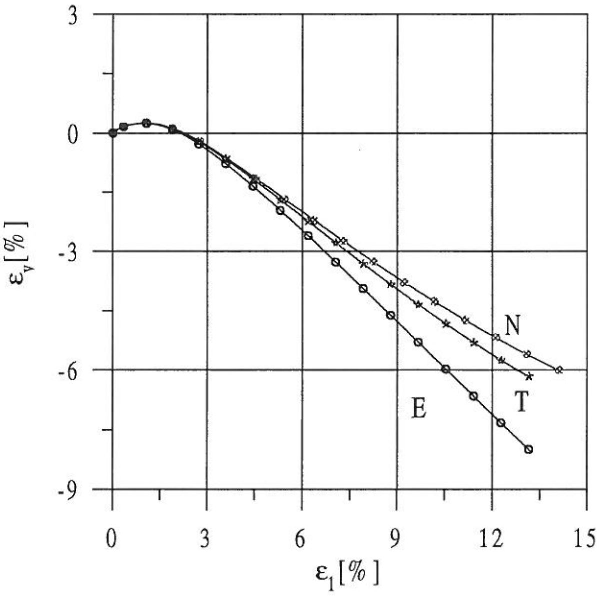

Fig. 8. Volumetric strain versus axial engineering and natural strain.

The three curves representing each of the three methods diverge significantly as the axial strain increases. At failure the relative difference is as high as 18%.

This difference affects the angle of dilation, which is an important parameter in the description of the volumetric behaviour of soils. The value of the angle of dilatation is calculated to 12.7°, 13.6 and 18.1° based on the methods N, T and E, respectively.

4.2 *The single hardening model*

The single hardening model is an advanced constitutive model for frictional material such as soils, concrete and rock (Kim & Lade, 1988; Lade & Kim, 1988a,b and Lade & Nelson, 1987).

The single hardening model is a monotonic elasto-plastic constitutive model. The model consist as many other elasto-plastic models of a failure criterion, a yield criterion, a plastic potential and a hypoelastic model. The failure criterion determines the maximum load that a soil element can withstand. The yield criterion controls whether plastic deformations occurs. The plastic potential controls the direction of the plastic strain increments whereas the elastic model determines the elastic behaviour of the material.

The single hardening model follows a non-associated flow rule as the yield function and the function for the plastic potential are different functions. The model can in addition handle stress-strain behaviour in the softening regime, but cannot in the form used herein handle large stress reversals. The model is furthermore restricted to model the stress-strain behaviour of isotropic materials.

The single hardening model has as many as twelve material parameters, but they are all easily determined. This paper concerns the choice of two strain measures and their effect on some key geotechnical parameters. Therefore, it has been decided not to show any of the expressions involved in the model and just refer to the references and use an identical parameter representation.

Parameters listed in table 1 are calibrated on the basis of six conventional triaxial tests performed on Eastern Scheldt Sand deposit with a relative density of D_r=60% and constant confining pressures ranging from 80-800 kPa, (Jakobsen and Praastrup 1998).

Table 1. Material parameters

Parameter	T	E	N
Elastic behaviour			
ν	0,20	0,20	0,20
M	477,65	477,65	458,45
λ	0,4142	0,4142	0,4081
Failure Criterion			
$a^{\dagger}$	0,00	0,00	0,00
$m^{\dagger}$	0,2879	0,2879	0,2879
$\eta_1^{\dagger}$	70,19	70,19	70,19
Plastic Potential			
$\psi_1^{\dagger}$	0,00754	0,00754	0,00754
ψ_2	-3,1375	-3,1118	-3,1540
μ	1,9862	1,7814	2,0611
Yield Function			
$10^{-4}C$	1,3101	1,3101	1,2748
p	1,6188	1,6188	1,6078
h	0,6416	0,6476	0,6166
α	0,5613	0,5726	0,5525
† Strain independent parameters			

Material parameters fitted solely on the basis of stresses are independent of the three methods as explained above. Poisson's ratio ν is set to a constant value of 0.2 due to significant scatter in the test results, (Lade and Nelson 1987).

The variation among the parameters associated with the elastic behaviour of the material is small. The parameters for T and E are identical. A minor change in the elastic parameters can barely be observed on a monotonic stress-strain curve as the elastic contribution is small compared to the plastic contribution as none of the specimens have been presheared.

Minor changes among the parameters included in the plastic potential and the yield function have a more pronounced effect on the overall stress-strain behaviour and is most conveniently illustrated by a prediction of the relationship between volume change and the axial displacement, see figure 9.

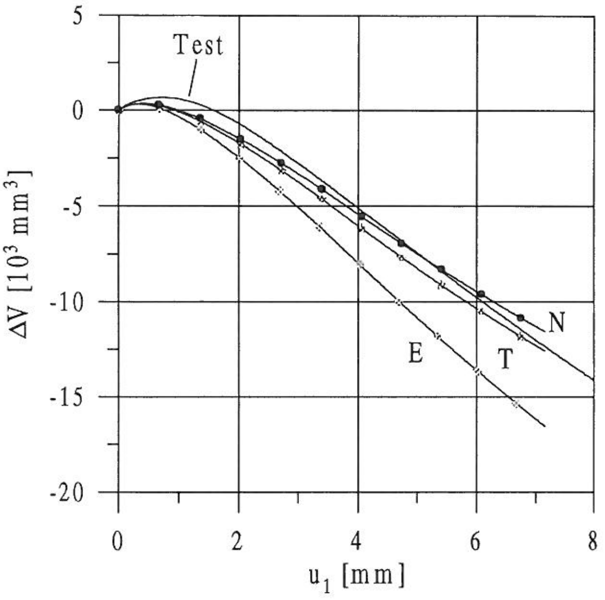

Fig. 9. Predicted and measured volume change versus axial displacement

Figure 9 shows that the three methods have a considerable impact on the predictions. It is, moreover, observed that none of the predictions captures the compressive portion of the measured soil response. This may be a shortcoming of the single hardening model itself and has nothing to do with the three methods.

Figure 9 shows further that the difference between the three methods at small levels of axial displacement is insignificant. It is also observed that the method E fails in predicting the soil response at moderate to high displacements levels. This is a consequence of limitations associated with this method.

The methods N and T captures the soil response equally well. As method N is theoretically consistent and methods T is theoretically inconsistent, the correct choice is to use method N is the analysis of triaxial tests.

A comparison of the graphs in figure 8 and figure 9 reveals that the single hardening model is very robust and that it can be used independently of the two strain measures. So the variation among the parameters in table 1 reflects the difference between the three methods. Hence, allowing the model to capture the soil response using different strain measures.

5 CONCLUSION

Within the scope of this work, which concerns the choice of strain measures in geomechanics, the conclusion that can be drawn upon the results presented in this paper is that the chosen strain measure has a considerable effect on the volumetric strain. The effect on the volumetric strain affects strain-dependent geotechnical parameters, while parameters solely determined on the basic of stresses are unaffected. The traditional analysis of triaxial tests, method T, has been found to be inconsistent with the theory of infinitesimal deformations, as a infinitesimal strain measure is mixed with a finite strain measure. Therefore, two theoretically consistent methods, denoted by N and E, respectively, were proposed and examined.

The methods E has been found to produce erroneous results within the normal range of deformations in both triaxial and true triaxial tests. Therefore, the authors suggest that the method N is used in the analysis of triaxial and true triaxial tests. The method E could however apply in situation where deformations are reasonably small and where the material with reasonable justification could be modelled as a purely elastic material. The recommended procedure for analysing triaxial tests is outlined below:

- Establish the exact displacement field using measured volume change and measured axial displacement.
- Establish all Cauchy stress components using the current cross sectional area.
- Choose an appropriate strain measure using E or N and calculate all strains accordingly.
- Finally display the results using the same diagrams as used in T.

The intention behind this paper was to make people interested in geomechanics aware of that the traditional analysis of triaxial tests is theoretically inconsistent. It is the authors hope that this paper will set a new standard for analysing triaxial tests.

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